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TURBULENT MIXING OF GAS FLOWS IN THE PRESENCE OF A PRESSURE GRADIENT

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16. Abstract

An examination is made of turbulent mixing of homogeneous gas flows in the presence of a longitudinal pressure gradient. The equations of a turbulent boundary layer on the assumption of a Prandtl mixing path reduce to the "laminar" form. The calculated dependence of velocity at the separating line on the Mach number at the upper boundary of the mixing zone M_e is compared with earlier-published solutions.

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G. P. Shinkin

An examination is made of turbulent mixing of homogeneous gas flows in the presence of a longitudinal pressure gradient. The equations of a turbulent boundary layer on the assumption of a Prandtl mixing path reduce to the "laminar" form. Variable similarities are introduced in which the considerations of local similarity formulated in [1] are extended to this case. The calculated dependence of velocity at the separating line on the Mach number at the upper boundary of the mixing zone M_e for isoenergetic mixing with a quiescent gas is compared with the solutions of Korst [2], Abramovich [3], and the Nash approximation [4].

1. Let us examine the turbulent mixing of two homogeneous gas flows in the presence of a pressure gradient. In the usual assumptions on the boundary layer and with the molecular effects neglected, the equations of momentum, continuity, and energy for the turbulent zone of mixing are of the form [5]

$$\rho u \frac{\partial u}{\partial x} + \rho v_* \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial \tau_T}{\partial y}$$

$$\frac{\partial(\rho u r_0^k)}{\partial x} + \frac{\partial(\rho v_* r_0^k)}{\partial y} = 0 \qquad (1.1)$$

$$\rho u \frac{\partial i}{\partial x} + \rho v_* \frac{\partial i}{\partial y} = u \frac{dp}{dx} + \frac{\partial q_T}{\partial y} + \tau_T \frac{\partial u}{\partial y}$$

Here x and y are the distances along the separating flow line and with respect to the normal to it, r_a is the distance from the axis of symmetry to the separating flow line, and k takes on the values 0 and 1 for the plane and axisymmetric cases, respectively

$$\tau_T = -\rho\langle u'v'\rangle, \quad q_T = -\rho\langle i'v'\rangle, \quad v_* = v(1 + \langle\rho'v'\rangle / \rho v)$$

and u, v, ρ, p, and i are the velocity components with respect to the x and y axes, density, pressure, and enthalpy -- all quantities being averaged over a sufficiently long time interval -- (the angled brackets also denote averaging over time).

The equation of state for the averaged quantities can be represented, approximately, as [5]

$$p / \rho i = (\gamma - 1) / \gamma$$

where γ is the ratio of heat capacities (we assume γ = const; in the calculations γ was assumed equal to 1.4).

The boundary and initial conditions are given as follows:

$$
\begin{array}{ll}
u \to u_e(x), \quad i \to i_e(x) & \text{for } y \to \infty \\
v_* = 0 & \text{for } y = 0 \qquad (1.2) \\
u \to u_{-e}(x), \quad i \to i_{-e}(x) & \text{for } y \to -\infty \\
u = f_1(y), \quad v_* = f_2(y), \quad i = f_3(y) & \text{for } x = 0
\end{array}
$$

where the subscripts e and -e correspond to the parameters at the upper and lower bounds of the mixing zone, and f_1, f_2, and f_3 are given functions.

2. To express the turbulent friction and heat flow, let us use the generalized hypothesis of the Prandtl mixing path

$$\tau_T = \rho l_u l_v \left|\frac{\partial u}{\partial y}\right| \frac{\partial u}{\partial y} = A_\tau \frac{\partial u}{\partial y}$$

$$q_T = \rho l_i l_v \left|\frac{\partial u}{\partial y}\right| \frac{\partial i}{\partial y} = A_q \frac{\partial i}{\partial y}$$

The turbulent Prandtl number is

$$\mathrm{Pr}_T = A_\tau / A_q = l_u / l_i$$

In the case of free turbulence, these formulas are considerably simplified (and specifying the Prandtl formula [5])

$$\tau_T = \varkappa\rho b(x)\left[u_e(x) - u_{-e}(x)\right]\frac{\partial u}{\partial y}$$
$$q_T = \varkappa\frac{\rho b(x)}{\mathrm{Pr}_T}\left[u_e(x) - u_{-e}(x)\right]\frac{\partial i}{\partial y} \qquad (2.1)$$

Here κ is a dimensionless quantity determined experimentally, b(x) is some effective width of the mixing zone, and Pr_T is also a quantity determined experimentally; we will assume that the number Pr_T does not depend on the coordinates, but can depend, for example, on i.

Further, substituting relations (2.1) into the system (1.1) and introducing the new variable

$$x_1 = \int_0^x \varkappa b(x)\left[u_e(x) - u_{-e}(x)\right]dx$$

we have

$$\rho u\frac{\partial u}{\partial x_1} + \rho v_{*1}\frac{\partial u}{\partial y} = -\frac{dp}{dx_1} + \frac{\partial}{\partial y}\left(\rho\frac{\partial u}{\partial y}\right)$$
$$\frac{\partial(\rho u r_0^k)}{\partial x_1} + \frac{\partial(\rho v_{*1} r_0^k)}{\partial y} = 0$$
$$\rho u\frac{\partial i}{\partial x_1} + \rho v_{*1}\frac{\partial i}{\partial y} = u\frac{dp}{dx_1} + \frac{\partial}{\partial y}\left(\frac{\rho}{\mathrm{Pr}_T}\frac{\partial i}{\partial y}\right) + \rho\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.2)$$
$$(v_{*1} = v_* / [\varkappa b(u_e - u_{-e})])$$

The boundary and initial conditions (1.2) remain unchanged. In the new variables (x_1, y) the system of equations (2.2) is very similar to the corresponding system in the laminar case, only instead of viscosity and Prandtl number, ρ and Pr_T will be used.

To investigate the question of the existence of the automodeling solution, let us introduce the variables

$$\xi = \int_0^{x_1} \rho_e^2 u_e r_0^{2k}\,dx_1, \qquad \eta = u_e r_0^k \int_0^y \rho\,dy$$

Further, let us write out system (2.2) in the von Mises variables (ξ, ψ) such that $\dfrac{\partial\psi}{\partial\eta} = u_1, \quad \dfrac{\partial\psi}{\partial\xi} = -v_1$

$$\left(u_1 = u/u_e, \; v_1 = \left(u \frac{\partial \eta}{\partial \xi} + \frac{\rho v_{*1}}{\rho_e^2 r_0^k} \right) \Big/ u_e \right)$$ /155

and assuming that the solution depends on a single variable $\zeta = \psi/\sqrt{\xi}$, we get

$$-\frac{\zeta}{2} u_1 \frac{du_1}{d\zeta} = \frac{du_e}{d\xi} \frac{\xi}{u_e} \left[\frac{\rho_e}{\rho} - u_1^2 \right] + u_1 \frac{d}{d\zeta} \left(u_1 i_1^{-2} \frac{du_1}{d\zeta} \right)$$

$$-\frac{\zeta}{2} \frac{di_1}{d\zeta} = \frac{d}{d\zeta} \left(\frac{u_1 i_1^{-2}}{\Pr_T} \frac{di_1}{d\zeta} \right) + M_e^2 (\gamma - 1) u_1 i_1^{-2} \left(\frac{du_1}{d\zeta} \right)^2 \qquad (2.3)$$

$$(i_1 = i/i_e)$$

With the boundary conditions

$$u_1 \to 1, \quad i_1 \to 1 \quad \text{as } \xi \to \infty$$

$$u_1 \to u_{-e}/u_e, \quad i_1 \to i_{-e}/i_e \quad \text{as } \xi \to -\infty \qquad (2.4)$$

In the general case, when a pressure gradient is present, the conditions of automodelity (they are the same as in the laminar case [1]) are not satisfied.

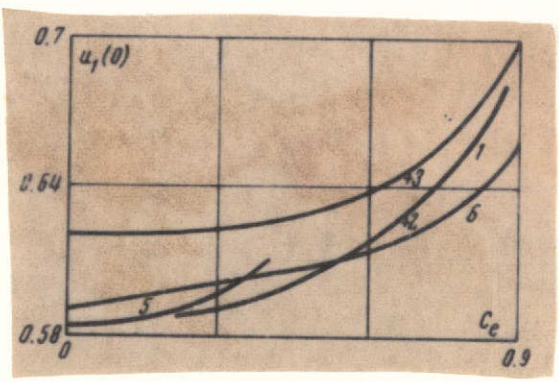

Fig. 1

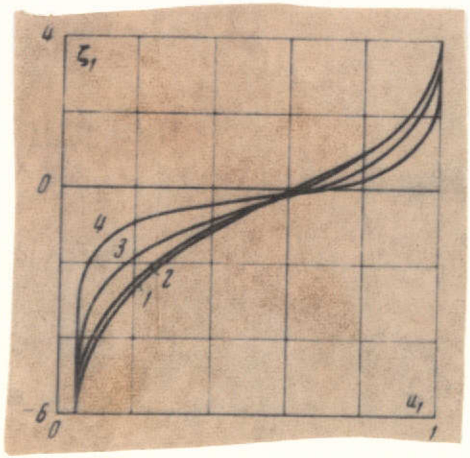

Fig. 2

A question of the possibility of using local similarity in this case arises. Here we note that equations (2.3) differ from the corresponding equations in the laminar case [1] only by the fact that here the enthalpy in the terms describing friction, heat flow, and dissipation of energy appears as i_1^{-2}, while in the laminar case -- as i_1^{-n}, where n is the exponent in the viscosity law: $\rho\mu = c\rho i^{-n}$ $(n \sim 0.3)$.

Thus, the results of the paper [1] on the minor effect of the term $(du_e/d\xi)(\xi/u_e)\cdot(\rho_e/\rho - u_1^2)$ can probably be extended also to the turbulent case, but it should be noted that here the conditions for the realization of local similarity for velocity are less favorable, since the profile u_1 depends more substantially on the enthalpy profile i_1, and therefore, also on the Mach number $M_e = M_e(x)$. In spite of this, it can still be anticipated that in practice local similarity for not overly large pressure gradients yields acceptable accuracy, that is, the profiles $u_1(\zeta)$ and $i_1(\zeta)$ will be quite closely described by the automodeling profiles for the same boundary conditions (2.4) and Mach numbers in problems with or without the pressure gradient.

3. From the system (2.3) it is clear that in the turbulent case, as already noted above, the parameters M_e and i_{-e}/i_e must more strongly affect the automodeling velocity profile than in the laminar case. The dependence of velocity at the separating flow line on these parameters can be calculated, without involving any empirical data. Fig. 1 (curve 1) shows the dependence of velocity on the separating flow line $u_1(0)$ on the Croccot number

$$c_e = \left[1 + \frac{2}{(\gamma - 1) M_e^2}\right]^{-1/2}$$

for $p = \text{const}, \quad u_{-e} = 0, \quad i_{e0} = i_{-e0}$

where i_{e0} and i_{-e0} are the enthalpies of retardation of the upper /156
and lower flows. In the calculation Pr_T was assumed to be equal to 0.7; for $M_e = 2$ the velocity $u_1(0)$ was also considered for $Pr_T = 0.5$ toward 1 (points 2 and 3). The calculated dependence was calculated with the known solutions of Korst [2] and Abramovich [3], and the Nash approximation [4] (curves 4, 5, 6)

$$u_1^2(0) = 0.348 + 0.018 M_e$$

In the Korst solution $u_1(0) = 0.62$ for $M_e = 0$, which does not agree with the value $u_1(0) = 0.584$ obtained by Tollmien [6], therefore Nash also proposed the approximation $u_1(0) = 0.588$ for $M_e = 0.5$, which is in agreement with the Tollmien value. These results show that the system of equations (2.3) quite closely describes the nongradient turbulent mixing zone. Fig. 2 shows the automodeling profiles $u_1(\zeta_1)$ for the Mach numbers $M_e = 0.5$, 1, 2, and 4 (curves 1, 2, 3, and 4) for $u_{-e}/u_e = 0.05$, $t_{e0} = t_{-e0}$, and $Pr_T = 0.7$. We note that by appropriately superimposing the Hertler profile for an incompressible fluid (for example, by superimposing the velocity values at the separating flow line)

$$u_1(\zeta_1) = 1 - \frac{1}{2}\left(1 - \frac{u_{-e}}{u_e}\right)\left(1 - \frac{1}{\sqrt{\pi}}\int_0^{\zeta_1} \exp\left(-\frac{\zeta_1^2}{4}\right) d\zeta_1\right)$$

we can obtain a satisfactory description of the resulting automodeling velocity profiles. The variable ζ_1 is related to the earlier-introduced variable ζ by the relation $d\zeta = u_1(\zeta_1)\,d\zeta_1$.

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